

Paper Reference 9FM0/3A
Pearson Edexcel Level 3 GCE
Further Mathematics

Advanced
PAPER 3A: Further Pure Mathematics 1
Wednesday 18 June 2025 – Afternoon
Time: 1 hour 30 minutes

Question Paper

**THIS DOCUMENT MUST BE
RETURNED TO PEARSON AT THE END
OF THE EXAMINATION.**

In the boxes below, write your name,
centre number and candidate number.

Surname					
Other names					
Centre Number					
Candidate Number					

YOU MUST HAVE

**Mathematical Formulae and Statistical
Tables (Green), calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

**Candidates may use any
calculator permitted by Pearson
regulations. Calculators
must not have the facility for
symbolic algebraic manipulation,
differentiation and integration,
or have retrievable mathematical
formulae stored in them.**

Turn over

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 9 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Turn over

1. (a) Refer to the table for Question 1 in the Diagram Booklet.

Given that

$$y = e^{\operatorname{cosec}^2 x}$$

complete the table in the Diagram Booklet with the value of y corresponding to $x = 2$, giving your answer to 2 decimal places.
(1 mark)

(continued on the next page)

1. continued.

(b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to one decimal place, the value of

$$\int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} dx$$

(3 marks)

(continued on the next page)

Turn over

1. continued.

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_{1.5}^{2.5} e^{\cot^2 x} dx$$

(2 marks)

(Total for Question 1 is 6 marks)

Turn over

2. Refer to the information for Question 2 in the Diagram Booklet. It shows the Taylor series expansion of $f(x)$ about $x = a$

Given that

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2xy = 4 \quad (\text{I})$$

(continued on the next page)

Turn over

2. continued.

(a) show that

$$\frac{d^4 y}{dx^4} = a \frac{dy}{dx} + bx \frac{d^2 y}{dx^2} + c \frac{d^3 y}{dx^3}$$

where a , b and c are integers to be determined.

(4 marks)

(continued on the next page)

Turn over

2. continued.

Hence, given that

$$y = 1 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 2$$

- (b) determine a Taylor series solution, in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^4$, of the differential equation (I), giving each coefficient in simplest form. (4 marks)**

(Total for Question 2 is 8 marks)

Turn over

3. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the values of x for which

$$\frac{x^2 - 4}{|x - 5|} > 4x + 8$$

(6 marks)

(Total for Question 3 is 6 marks)

Turn over

4. $y = \sqrt{(15 + e^{2x})}$

(a) Show that

$$\frac{d^2y}{dx^2} = \frac{e^{2x}(e^{2x} + 30)}{(15 + e^{2x})^{\frac{3}{2}}}$$

(3 marks)

(b) Hence determine the Maclaurin series expansion for y , in ascending powers of x , up to and including the term in x^2 , giving each term in simplest form.

(2 marks)

4. continued.

- (c) Use the series expansion for $\sin x$ in ascending powers of x to determine, in simplest form, the first 2 non-zero terms in ascending powers of x of the series for $\sin 3x$
(1 mark)

(continued on the next page)

Turn over

4. continued.

(d) Use the answers to parts
(b) and (c) to show that

$$\lim_{x \rightarrow 0} \frac{\sqrt{(15 + e^{2x})} - 4}{\sin 3x} = \frac{1}{12}$$

(2 marks)

(Total for Question 4 is 8 marks)

Turn over

5. (a) Use the substitution

$$t = \tan\left(\frac{x}{2}\right) \text{ to show that}$$

$$\int \frac{1}{1 + \operatorname{cosec} x} dx = \int \frac{4t}{(1 + t^2)(1 + t)^2} dt$$

(3 marks)

(continued on the next page)

Turn over

5. continued.

(b) Hence show that

$$\int \frac{1}{1 + \operatorname{cosec} x} dx = Ax + \frac{B}{1 + \tan\left(\frac{x}{2}\right)} + k$$

where **A** and **B** are constants to be determined and **k** is an arbitrary constant.

(5 marks)

(Total for Question 5 is 8 marks)

Turn over

6. (i) Use L'Hospital's Rule to show that

$$\lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x \sin 9x} = \frac{49}{18}$$

(4 marks)

(continued on the next page)

6. continued.

(ii) $y = x^2 e^{3x}$

(a) Use Leibnitz's theorem to show that, for $k \in \mathbb{N}$

$$\frac{d^k y}{dx^k}$$

$$= 3^{k-2} e^{3x} (Ax^2 + Bkx + Ck(k-1))$$

where **A**, **B** and **C** are integers to be determined.
(5 marks)

(continued on the next page)

Turn over

6. (ii) continued.

(b) Hence determine the values of x for which

$$\frac{d^9 y}{dx^9} = 0$$

(2 marks)

(Total for Question 6 is 11 marks)

Turn over

7. The concentration, $P \text{ mg m}^{-3}$, of a pollutant in a reservoir, t days after the pollutant entered the reservoir, is modelled by the differential equation

$$\frac{1}{P} \frac{dP}{dt} = 4tP^2 - 1 \quad (\text{I})$$

- (a) Show that the transformation

$$x = \frac{1}{P^2} \text{ transforms equation (I)}$$

into the equation

$$\frac{dx}{dt} - 2x = -8t \quad (\text{II})$$

(3 marks)

(continued on the next page)

Turn over

7. continued.

Given that

$$P = 0.5 \text{ when } t = 0$$

(b) solve differential equation (II)
to show that, according to the
model,

$$P^2 = \frac{1}{4t + 2 + ke^{2t}}$$

where k is a constant to be
determined.

(6 marks)

(continued on the next page)

Turn over

7. continued.

Given that the concentration of the pollutant in the reservoir, 3 days after the pollutant entered the reservoir, was 0.034 mg m^{-3}

(c) comment on the reliability of the model, giving a reason for your answer.

(2 marks)

(Total for Question 7 is 11 marks)

Turn over

8. The ellipse **E** has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line **L** has equation

$y = mx + c$ where **m** and **c** are constants.

(continued on the next page)

8. continued.

(a) Show that the x coordinates of the points of intersection of E and L satisfy the equation

$$(16m^2 + 9)x^2 + 32cmx + 16c^2 + k = 0$$

where k is a constant to be determined.

(2 marks)

(continued on the next page)

Turn over

8. continued.

Hence, given that the line L

- is a tangent to E
- passes through the point $(10, 20)$

(b) determine the possible equations of L

(5 marks)

(Total for Question 8 is 7 marks)

Turn over

9. The parallelogram **P** has vertices **E**, **F**, **G** and **H**, with coordinates

$$\mathbf{E}(10, -1, -6)$$

$$\mathbf{F}(7, -2, 7)$$

$$\mathbf{G}(10, 2, 9)$$

$$\mathbf{H}(13, 3, -4)$$

- (a) Determine the exact area of **P**, giving your answer as a simplified surd.

(3 marks)

(continued on the next page)

Turn over

9. continued.

The line **L** passes through **E** and is perpendicular to the plane containing **P**

(b) Determine an equation for **L**
giving your answer in the form
 $(\underline{r} - \underline{a}) \times \underline{b} = \underline{0}$

where $\underline{a}$ and $\underline{b}$ are constant
vectors.

(2 marks)

(continued on the next page)

Turn over

9. continued.

Given that

- P is one face of a parallelepiped
- the opposite face of the parallelepiped lies in the plane Π
- the point $(25, -4, 13)$ lies in Π

(c) determine the volume of the parallelepiped.

(5 marks)

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER